Variable Rate Modelling of Fluvial Thermal Erosion

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Abstract

In periglacial regions, frozen river banks are affected by thermal and mechanical erosion. In Siberia, bank retreat of up to 40 m per year are observed. This thermal erosion occurs during a few weeks, at springtime, for high enough water temperatures and river discharges. Until now, models of thermal erosion are based on the assumption of a constant melting rate. We have developed a more general model at variable rate, whose solution is calculated with the integral method. Results of this model are compared with experiments, carried out in a cold room. The model has contributed to better understand the roles of each parameter during the thermal erosion process. The duration of such acceleration phase is systematically studied.

Keywords: thermal erosion/ablation/permafrost/periglacial river/phase change/heat balance integral method

Introduction

Most periglacial rivers exhibit a breakup period and a flood season associated with a high discharge rates and high water temperatures. The Lena River in Siberia and its tributaries can be divided in to two classes (1) the Lena basin outlet and (2) the southern sub-basins (Aldan, upper Lena, Vilui valley). A relatively low stream temperature variation and a high discharge variation characterize the Lena basin outlet. The stream temperature varies from 0 °C to 14°C and the discharges can reach 100 000 m^3/s in early June (Gautier et al., 2003 ; Liu et al., 2005 and Yang et al., 2002). In the second case, the southern sub-basins, are characterized by relatively high stream temperature and low discharge. For these rivers, stream temperatures are up to 4°C higher than those over the Lena outlet (Liu et al., 2005) and can reach 18 °C and the discharges are about ten times smaller than those in the Lena basin outlet (Liu et al., 2005 and Yang et al., 2002). During the break-up and flood seasons, the water flow in permanent contact with frozen river banks induces both a fluvial thermal and mechanical erosion. This problem is non-linear because it involves a moving boundary (interface between solid and liquid) whose location is unknown. Until now, models of thermal erosion based on the assumption of a constant melting rate for a constant convective flux at the interface have been used to study thermal erosion of ice and permafrost (Costard et al., 1999). The objective of this study is to propose a model of thermal erosion of permafrost without the simplified assumption of a constant melt rate. Once validated by the comparison with laboratory experiments, the model will be applied to the Siberian rivers case.

The Ablation Model at Variable Rate

Mathematical model

A semi-infinite permafrost sample initially at a uniform negative temperature (T_∞) is suddenly heated by a turbulent water flow of permanent temperature (T_L) and discharge (Q_L) (Fig. 1).

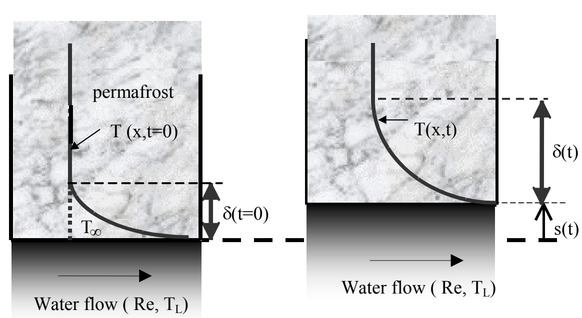

Figure 1. (a) The initially isothermal permafrost (T_∞) is suddenly heated by a permanent water flow (Re, T_L). After a rapid transient phase, the interface reaches the melting temperature (T_m=0°C). Then, the thermal erosion initiates and the interface starts to move. (b) The interface progressively moves. Its instantaneous position and erosion rate are given by s(t) and ds/dt. The instantaneous thickness of the thermal boundary layer is δ(t).

We suppose that all the sediment is immediately swept away by mechanical action of the water flow. The heat transfer occurs by conduction inside the permafrost (eq. 1). At the permafrost-water interface, the convective heat flux from the water flow must balance the latent heat absorbed by melting added to the conductive heat flux in the solid (eq. 2).

$$\frac{\partial T}{\partial t} = \left(\frac{k}{\rho . c_p}\right) \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) \qquad x \geq s(t) \qquad (1)$$

$$h(T_L - T_m) = \rho L \frac{\partial s}{\partial t} - k\left(\frac{\partial T}{\partial x}\right)_{x=s} \qquad x = s(t) \qquad (2)$$

where k [$W.m^{-1}.K^{-1}$], ρ [$kg.m^{-3}$], c_p [$J.kg^{-1}.K^{-1}$] and L [J/kg] are the thermal conductivity, the density, the specific heat and the latent heat of melting of the permafrost and h [$W.m^{-2}.s^{-1}$] is the heat transfer coefficient between turbulent water and permafrost.

Resolution by the heat balance integral method

The "heat-balance integral method" consists of integrating the heat conduction equation (eq. 3) over the thermal layer δ(t). It is based on the assumption of a quadratic boundary layer temperature (eq. 4).

$$\frac{\partial}{\partial t}\int_s^{\delta} T dx = \left(\frac{k}{\rho . c_p}\right)\left[\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=\delta} - \left.\frac{\partial T(x,t)}{\partial x}\right|_{x=s}\right] \qquad (3)$$

$$T(x,t) = T_\infty\left[-2\left(\frac{x-s}{\delta - s}\right) + \left(\frac{x-s}{\delta - s}\right)^2\right] \quad s(t) < x < \delta(t), \quad (4)$$

The temperature distribution (eq. 4) is substituted in equation (1) and the heat equation is integrated (eq. 3), applying boundary and initial conditions. Then, solutions in a-dimensional form are obtained (Goodman, 1958). The system of Goodman (1958) is solved for every time t during the thermal erosion process in order to get the instantaneous thermal boundary layer δ(t) and eroded thickness s(t) (Randriamazaoro et al., 2007).

Results

Both the thermal boundary layer and the ablation rate increase (acceleration phase) and stabilize (stationary phase) towards asymptotic values (eq. 5 and eq. 6) which correspond to the solutions of the constant rate-melting model (Aguirre et al., 1994 ; Costard et al., 2003).

$$\delta_{lim} = \frac{2.k.L}{c_{p.}.q_{conv}} \qquad (5)$$

$$\left(\frac{ds(t)}{dt}\right)_{lim} = \frac{q_{conv}}{\rho L + \rho . c_{p.}(T_\infty - T_m)} \qquad (6)$$

where q_{conv} is the heat flux exchanged by convection at the interface between the water flow and the permafrost and includes both the effects of water temperature and discharge (eq. 7).

$$q_{conv} = h.(T_L - T_f) \qquad (7)$$

Our model is validated by measurements of instantaneous eroded thickness of ice and permafrost samples in contact with a turbulent water flow in a cold room (Costard et al., 2003). One set of experiments are done on ice samples at an initial temperature equal to -7.5°C and the water temperature remains at 5.5°C during the whole experiments. Different Reynolds numbers are investigated. The ablation rate increases with the Reynolds number (Fig. 2).

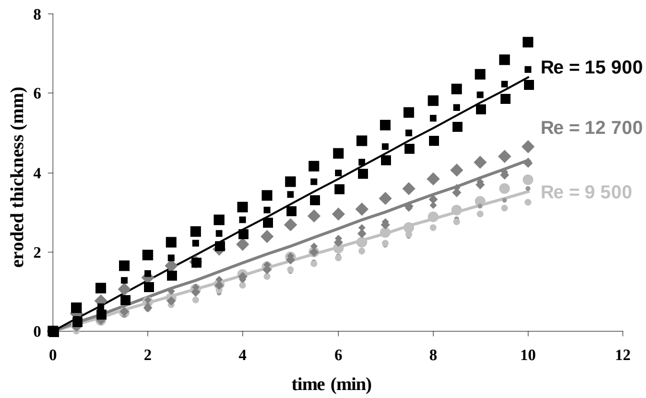

Figure 2. Measurements of eroded thickness of pure ice initially at –7.5°C, heated by a turbulent water flow at 5.5°C. In 10 minutes, the measured eroded thickness of the ice sample are 3.5 mm, 4.5 mm, 6.5 mm for Re=9 500, 12 700 and 15 900 respectively.

Experiments suggest that the eroded thickness increases linearly with time (Fig. 2). Considering a stationary erosion rate, the measured erosion rate of ice are respectively equal to 0.35 mm/min, 0.45 mm/min and 0.65 mm/min for Re=9 500, 12 700 and 15 900 respectively. We tested our model to these particular experimental conditions (Fig. 3). The model predicts a first phase of acceleration during which both the thermal boundary layer and the erosion rate increase (Fig. 3). This first phase lasts a few minutes. The asymptotic values of the calculated ablation rate (Fig. 3) are consistent with the measured values (Fig. 2).

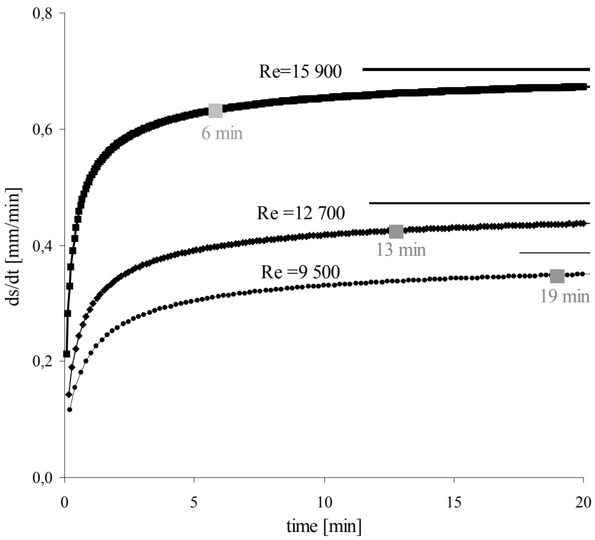

Figure 3. Theoretical erosion rate as a function of time for a water temperature equal to 5.5°C, an ice temperature equal to 7.5°C and different Reynolds numbers (Re=9 500, 12 700 and 15 900) applied to the geometry of our hydraulic channel. The erosion rate increases rapidly (acceleration phase) and stabilises (stationary phase). The duration of the acceleration phase is calculated from the time necessary to reach 90% of the asymptotic value. The greater the Reynolds number, the greater the erosion rate and the smaller the duration of the acceleration phase. Measured values (Fig. 2) are consistent with the asymptotic calculated values.

Other experiments are done on frozen ice sand samples (initially at –7.5°C) with different massic ice contents (ω=20%, 80%) and are compared with pure ice (Fig. 4). The turbulent water flow remains at 5.5°C and the Reynolds number is equal to 15 900.

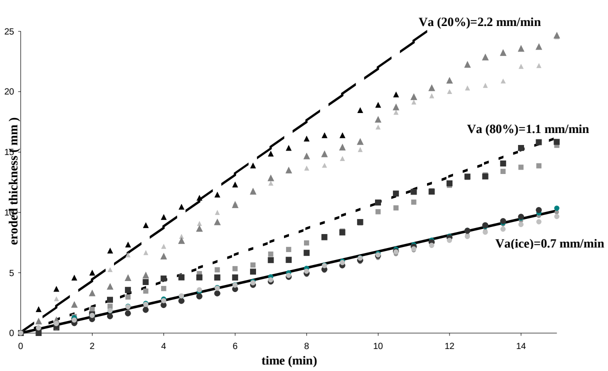

Figure 4. Measurements of eroded thickness of pure ice and sandy permafrost with different ice contents, initially at –7.5°C, heated by a turbulent water flow (5.5°C and Re=15 900). In 10 minutes, the measured eroded thickness are about 6 mm, 11 mm, 18 mm for pure ice, permafrost with ω=80% and ω=20% respectively. Linear fit gives measured erosion rates equal to 0.7 mm/min, 1.1 mm/min and 2.2 mm/min for pure ice, permafrost with ω=80% and ω=20% respectively.

The model is applied to these particular conditions and again the experiments and the model are consistent (Fig. 4 and Fig. 5).

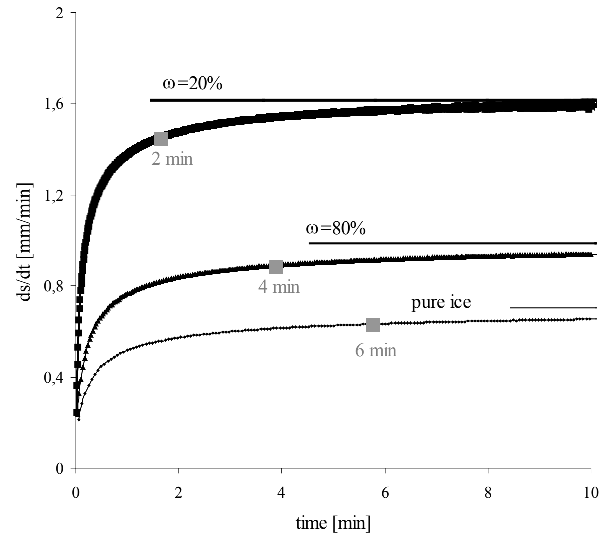

Figure 5. Theoretical erosion rate as a function of time for a water temperature equal to 5.5°C, an ice temperature equal to 7.5°C a Reynolds number equal to 15 900, applied to the geometry of our hydraulic channel. The erosion rate increases rapidly (acceleration phase) and stabilises (stationary phase). The smaller the ice content, the greater the erosion rate and the smaller the duration of the acceleration phase. Measured values are consistent with the asymptotic values.

The effects of the water temperature and the ice temperature are also investigated. The effects of the water temperature are predominant whereas the effects of the ice temperature are very weak. The erosion rate increases with the water temperature, the ice temperature, the Reynolds number and decreases with the ice content. The model at variable rate predicts a first acceleration phase whose duration is typically greater for smaller erosion rates.

Then the model is applied to the Lena river. The convective heat flux is calculated (eq. 7) for water temperature between 0°C and 20°C and for water discharges between 0 m^3/s and 120 000 m^3/s. The heat transfer coefficient h is estimated, using the empirical law of Lunardini (1986) (A=0.0078, α=0.3333, β=0.9270), and the Manning equation applied to the geometry of the channel (eq. 8) (Costard et al., 2003).

$$h = A\left[\left(\frac{\sqrt{s}}{n}\right)^{3/5} l^{\,3/5-\beta}\right]\left[\Pr^{\alpha}\frac{k_w}{\upsilon_w^{\ \beta}}\right]Q^{\,\beta-3/5} \qquad (8)$$

where S=0.0001 m/m, n=0.1, l=10 km, k_w, ν_w, Pr are the longitudinal slope, the Manning roughness coefficient, the width of river, the thermal conductivity of water [$W.m^{-1}.K^{-1}$], the cinematic viscosity of water [$m^2.s^{-1}$], and the Prandtl number, respectively.

Isoflux lines are plotted from the calculated values of the convective heat flux (Fig. 6). On this diagram, simultaneous measurements of water temperatures and discharges in the Lena basin outlet and the southern Lena sub-basins (Yang et al., 2002 ; Liu et al., 2005) every 10 days during the flood season are reported.

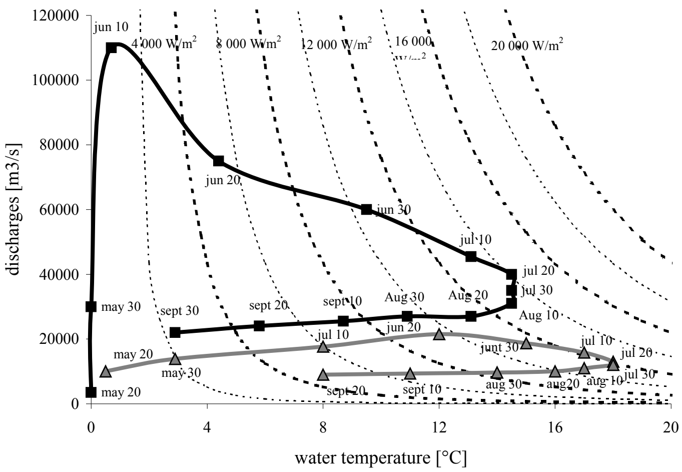

Figure 6. Diagram of heat flux versus discharge and water temperature.
---------- Isoflux lines (2 000 – 20 000 W/m²)
■ Water mean temperatures and discharges in the Lena basin outlet (Yang et al., 2002 ; Liu et al., 2005)
Δ Water mean temperatures and discharges in the Lena sub-basins (Aldan, Upper Lena, Vilui basin (Liu et al., 2005)).

As expected, the heat flux increases when the water temperature and the discharge increase simultaneously from May to mid-June. By contrary, from June to July, the water temperature is still increasing while the discharge decreases by about 50% and a positive trend of the heat flux is still observed. The maximum (12 000 W/m² and 14 000 W/m² for the southern Lena sub basin and for the Lena basin outlet, respectively) of the heat flux occurs during July when water reaches its maximum temperature. It appears that the convective heat flux evolution mainly depends on the water temperature evolution for Siberian rivers, during the flood season. The heat flux variation for the Aldan, Upper Lena, Vilui river is similar to the one of the Lena basin outlet. The relative higher values of water temperatures for the southern Lena sub-basin are compensated by the relative higher values of the discharges for the Lena basin outlet.

With regards to specific heat flow, the application of the model of variable rate on pure ice or permafrost should allow determination of the thermal boundary layer thickness (Fig. 7), the ablation rate and the duration of the acceleration phase (Fig. 8).

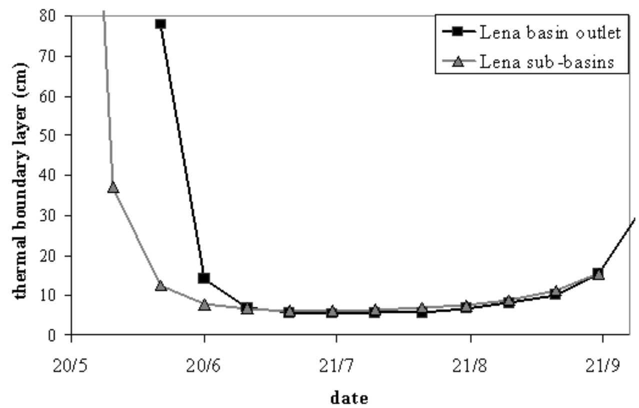

Figure 7. Evolution of the calculated thermal boundary layer thickness for the Lena basin outlet and the Lena sub-basins.

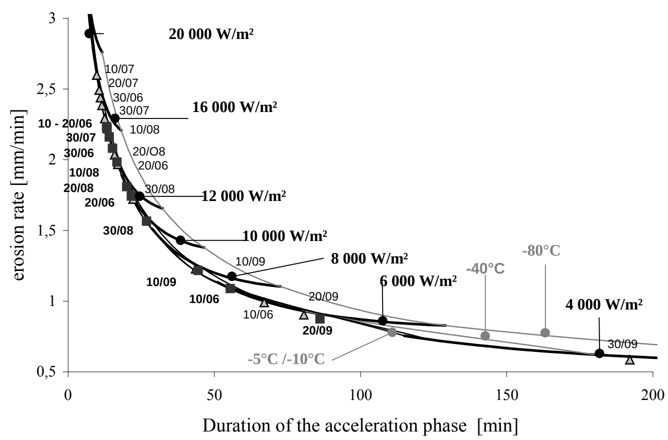

Figure 8. Diagram of ablation rate and duration of acceleration phase for various convective heat flux and ice temperature) –
▬▬▬ Isoflux lines (4 000 – 20 000 W/m²)
——— Isothermal lines (-10 to -80°C)
□ Ablation rate and duration of the acceleration phase in the Lena basin outlet
Δ Ablation rate and duration of the acceleration phase in the Lena tributaries (Aldan, Upper Lena, Vilui basin).

The greater the convective heat flux, the greater the erosion rate and the smaller the duration of the acceleration phase. The most favorable conditions to get the longer acceleration phase are obtained for the smaller values of the erosion rate at the beginning (early May) or at the end (October) of the flood season. For example, considering a small thermal erosion rate 0,06 mm/min (early May), the acceleration phase should last about 14 days.

Conclusions

A model of the fluvial thermal erosion has been formulated at variable rate. This mathematical model has been applied to a typical frozen river bank in permanent contact with a turbulent water flow. The expressions of the instantaneous melting thickness, ablation rate and thermal boundary layer have been obtained by integral method and validated by experiments on ice sample. An acceleration phase occurs at the beginning of the process. The duration of this acceleration phase is quantified. Typically, the acceleration phase lasts longer for a low ablation rate. The ablation rate increases with water temperature, discharge and decreases with ice content. The effects of stream temperature and discharge can be represented by the convective heat flux. The heat flux along the river banks are quite identical for the Lena basin outlet and for the southern Lena basin because relative higher values of stream temperatures for the southern Lena sub-basin are compensated by the relative higher values of the discharges for the Lena basin outlet. From our studies, the stream temperature is a quite important parameter which controls the evolution of the erosion rate during the flood season. Further studies will take into account the possible effect of the global warming on the thermal erosion rate.

Acknowledgments

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